ISSN 2229-5518

Advancements and Applications of Ant Colony Optimization: A Critical Review

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Abstract— Ant colony optimization (ACO) is a technique for optimization that was introduced in the early 1990's. The inspiring source of ant colony optimization is the foraging behavior of real ant colonies. This behavior is exploited in artificial ant colonies for the search of approximate solutions to discrete and continuous optimization problems and to important problems in telecommunications, such as routing and load balancing. This paper presents a comprehensive review on some of the advancements occurring in the domain of ant colony optimization and provides a possible classification based on the developments that took place. The merits of the advanced ACO algorithms as well as their applications are also discussed.

Index Terms— Ant Colony Optimization (ACO), ant system (AS), meta-heuristic, min-max ant system (MMAS), job shop scheduling problem (JSSP), travelling salesman problem, train scheduling problem, vehicle routing problem (VRP), Proportional-Integral-Derivative (PID).

1 Introduction

ANT Colony Optimization (ACO) is a recently developed, population-based approach which has been successfully applied to several combinatorial optimization problems. As the name suggests, ACO has been inspired by the behavior of real ant colonies, in particular, by their foraging behavior. One of its main ideas is the indirect communication among the individuals of a colony of agents, called (artificial) ants, based on an analogy with trails of a chemical substance, called pheromone, which real ants use for communication. The (artificial) pheromone trails are a kind of distributed numeric information which is modified by the ants to reflect their experience accumulated while solving a particular problem.

Andrias Baur *et al.* developed an ant colony optimization approach in the problem of scheduling job operations in the given number of available machines [1]. A MIN-MAX ant system was introduced by Holger H. Hoos *et al.* to exploit more strongly the best solutions found during the search and to direct the ants' search towards very high quality solutions and to avoid premature convergence of the ants' search [2]. A meta-heuristic method of ACO was proposed by John E. Bell *et al.* to an established set of vehicle routing problem (VRP). Modifications are made in the original ACO algorithm in order to allow the search of the multiple roots of the VRP [3]. An internet based peer-to-peer grid infrastructure was developed by Francesco Palmieri based on ant colony framework. Basically it is a problem on resource scheduling the main characteristics of this architecture developed are highlighted by its promising performance and skill ability, and its adaptive resource management and scheduling mechanism [4]. With the help of travelling salesman problem, a train scheduling problem was determined by K. Sankar using ant colony system meta-heuristic. A hierarchical process of rail transport planning was introduced and the ants' behavior which gave inspiration for ants' algorithm was presented [5].

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An urban traffic control system was developed by R. Foroughi *et al.* using modified ant colony optimization approach. The modified approach is based on the design of intelligent data routers and intelligent data mining [6]. The conventional approach of ACO was combined with taboo search by Kuo-Ling Huang *et al.* for job shop scheduling problem. The taboo search algorithm is embedded to improve the solution quality [7]. Marco Dorigo *et al.* depicted an overview and the advancements occurring in ACO and enumerated the applications of ACO as well [8]. Skudai *et al.* described the uses of soft computing techniques for edge detection [9]. A hybrid ant colony optimization approach for solving multi-objective design optimization of air cored solenoid was developed by Waiel F. Abd El Wahed *et al.* The proposed approach defers from the traditional one in its design of a multi-pheromone ant colony optimization as well as inclusion of steady state genetic algorithm and local search approach [10]. C. Naga Raju *et al.* presented a practical approach to detect the edges of noisy image pattern automatically and proposed a method using fuzzy rules which satisfies Markov symmetric property. ACO proposed as a tool for regularization in the noise images [11]. A hybrid ACO – Direct Cover (DC) technique was introduced by Mostafa Abd-El-Baar to synthesize the multi-level Multiple-Valued Logic (MVL) functions. ACO is used to decompose the MVL function into a number of levels and synthesize sub-function using a DC-based technique [12]. Vinay Chopra *et al.* suggested an ant colony optimization algorithm to solve Field Programmable Gate Array (FPGA) routing in design architecture with minimum number of tracks per channel. This developed ACO algorithm takes lesser amount of time and minimum channel width to route a FPGA chip [13]. K. M. Senthil Kumar *et al.* developed an ACO algorithm for Makespan minimization on unrelated parallel machines due to scheduling problem and to minimize the completion time of jobs [14]. Li-Ning Xing *et al.* developed a hybrid ant colony optimization algorithm for the extended capacitated arc routing problem which utilizes two kinds of heuristic information: arc cluster information and arc priority information, to guide the optimization process [15]. Ant

ISSN 2229-5518

colony optimization for location area planning in cellular networks was introduced by Ahmed Elwsishi *et al.* which outperform the Simulating Annealing approach [16]. M. Brignone *et al.* suggested a hybrid approach to inspect three dimensional homogeneous dielectric scatterers by using microwaves and combining the global optimization capabilities of ACO algorithm [17]. Section II presents the basic algorithm of Ant Colony Optimization while section III discusses the various classifications of the different advanced approaches of ACO as per literature. In section IV, the applications of the approaches discussed over the original algorithm are summed up with a view to used in future for different engineering applications.

2 Ant Colony Optimization Algorithm

A colony of artificial ants cooperates to find good solutions, which are an emergent property of the ant's co-operative interaction. Based on their similarities with ant colonies in nature, ant algorithms are adaptive and robust and can be applied to different versions of the same problem as well as to different optimization problems. The main traits of artificial ants are taken from their natural model. These main traits are artificial ants exist in colonies of cooperating individuals, they communicate indirectly by depositing pheromone they use a sequence of local moves to find the shortest path from a starting position, to a destination point they apply a stochastic decision policy using local information only to find the best solution.

In most application the amount of pheromone deposited is proportional to the quality of the move an ant has made. Thus the more pheromone, the better the solution found. After an ant has found a solution, it dies; i.e.it is deleted from the system.

ACO is depending upon the pheromone matrix $\tau = \{\tau_{ij}\}$ for the construction of good solutions. The initial values of τ are

set $\tau_{ij} = \tau_0 \forall (i, j)$, where $\tau_0 > 0$.

The probability $P_{ij}^A(t)$ of choosing a node j at node is defined in the equation (1). At each generation of the algorithm, the ant constructs a complete solution using this equation, starting at source node.

$$P_{ij}^A(t) = \frac{[\tau_{ij}(t)]^\alpha [\eta_{ij}]^\beta}{\sum_{ij \in T^A} [\tau_{ij}(t)]^\alpha [\eta_{ij}]^\beta}; i, j \in T^A \quad \text{... (1)}$$

where η_{ij} represents the heuristic function.

α and β = constants that determine the relative influence of the pheromone values and the heuristic values on the decision of the ant.

T^A = the path effectuated by the ant A at a given time.

The quantity of pheromone $\Delta\tau_{ij}$ on each path may be defined as

$$\Delta\tau_{ij}^A = \begin{cases} \dfrac{L^{\min}}{L^A} & \text{if } j \in T^A \\ 0 & \text{else} \end{cases} \quad \text{... (2)}$$

where L^A = the value of the objective function found by the ant A.

L^{min} = the best solution carried out by the set of the ants until the current iteration.

The pheromone evaporation is a way to avoid unlimited increase of pheromone trails and also it allows the forgetfulness of the bad choices.

$$\tau_{ij}(t) = p\tau_{ij}(t-1) + \sum_{A=1}^{NA} \Delta\tau_{ij}^A(t) \quad \text{... (3)}$$

where NA = number of ants.
p = the evaporation rate $0 < p \le 1$.

Implementation Algorithm
Step I Initialize the pheromone trail and the heuristic value.
Step II Place the ant on the node. Compute the heuristic value associated on the objective (minimize the error).

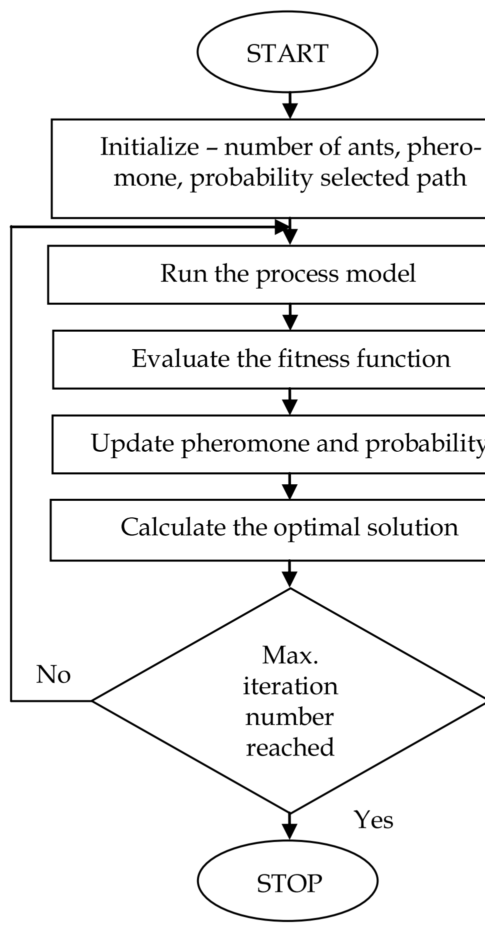

Figure 1: Flow chart of ACO Algorithm

Step III Use pheromone evaporation given by equation (3) to avoid unlimited increase of pheromone trails and allow the forgetfulness of bad choices.
Step IV Evaluate the obtained solutions according to the objectives.
Step V Display the optimum values of the optimization parameters.
Step VI Update the pheromone, according to the optimum solutions calculated at step V. Iterate from step II until the maximum of iterations is reached [18].

3 Classifications Based On Techniques

3.1 Hybridizations of ACO Techniques

The proposed hybrid technique produces more efficient realizations; the algorithm using ACO can decompose a given function into a number of sub-functions. A hybrid ACO algorithm with taboo search algorithm can be applied for job shop scheduling problem (JSSP). A taboo search algorithm is embedded to improve the solution quality. ACO with fast taboo algorithm, this proposed a global pheromone update queue with good schedules to update the pheromone trails diversely and history information can be utilized more effectively.

This approach exploits the strengths of the two methods i.e. the high computational efficiency of the linear sampling method (LSM) and the global optimization capabilities of the ACO. The approach can also be characterized by the exploitation of heuristic information, adaptive parameters and local optimization techniques. In the future research, the service architecture and service scheduling should be optimized synchronously using meta-heuristic.

Improving the multi-pheromone ant colony optimization by integration with steady state genetic algorithm and local search approach improve the quality of the solutions. For future work, the approach should be tested for more complex real-world applications [7, 10, 12, 15, 17].

3.2 Modified Approach

A new ant colony based optimizer shows a very good optimization path and have changed the original version of ACO. The modified algorithm can be used for applications such as designing intelligent data routers, intelligent data mining, etc. A modified ACO method can be used to select the optimum path from origin to destination. The ACO algorithm can be able to optimize two parameters and also able to initialize the number of ants with mentioned value and proving the good performance [6].

3.3 Meta-heuristic Method

By the use of meta-heuristic method of ant colony optimization (ACO), the algorithm is successful in finding solutions within 1% of known optimal solutions and the use of multiple ant colonies is found to provide a comparative solution technique for larger problems. Future research should focus on improving ACO algorithm for solving larger problems.

The ACO metaphor, a recently developed meta-heuristic has proven its potential for various combinatorial optimization problems. ACO has shown good results in the applications of job shop. Due to heuristic criteria and performing pair wise swap as a local search procedure yielded very good results for large problem instances and outperformed all leading heuristic approach.

ACO algorithm can also be applied for solving routing alternatives utilizing approach of hard combinatorial optimization problems. This suggested that the developed ACO algorithm is taking fewer amounts of time and minimum channel width. ACO algorithm takes less CPU time which is an optimal solution. ACO performed better and has routed the circuited with minimum channel width as compared to classical algorithms. This algorithm is more effective with less execution time and also proposed method achieved better solutions.

ACO can be interpreted as an extension of traditional construction heuristics which are available for many combinatorial optimization problems. It is noted that companies have started to use ACO algorithm for real-world applications.

The small and medium size problems were solved by using ant colony system (ACS) and compared with exact optimum solutions to check for quality and accuracy. The solutions showed that ACS has good quality and time savings.
Meta-heuristic techniques are often used for analyzing and solving practical sized instances. The potential improvement has been accomplished through the design and the analysis of ACO approach. The ACO approach outperforms the solution quality for a wide range of problem instances. However, further improvement to enhance the proposed approach is still needed [1, 3, 5, 8, 13, 16].

3.4 Population-based Approach

ACO is a recently developed, population-based approach which has been applied to several NP-hard (Non-deterministic Polynomial-time) combinatorial optimization problems. This exploits the best solutions during the search and ants search towards very high quality solutions. MMAS (MAX-MIN ant system) has a strong improved performance as compared to AS (Ant System). The introduction of MMAS is that the utilization of pheromone trail limits to prevent premature convergence; ultimately the performance and applicability of ACO algorithms can be further improved [2].

3.5 Architecture based on Swarm Intelligence

This architecture is based on swarm intelligence and ant colony meta-heuristic, to map the solution capability and resource the scheduling problem. The main characteristics of this architecture are by its promising performance and scalability. The proposed approach is based on swarm intelligence and precisely on the ACO meta-heuristic implemented in a multi-agent system scenario. Hence, the use of modular and extensible multi-agent system simplifies and improves the efficiency in the architecture development [4].

3.6 Advances of Soft Computing

As compared to fuzzy set theory, ANN and GA, ACO selects correct edges of the image. This method works relatively slow in comparison with other edge detection methods. ACO can act as an edge enhancement method and predicted to give astonishing response when applied on edges detection using a fuzzy method [9].

3.7 Improved ACO Technique

The normal ACO approach requires extensive computation so improved ACO algorithm was implemented which satisfied the Markov symmetric property. For textured and complex non-textured images, ACO produced thicker edges. Since, to produce thin edges, the fuzzy based ACO works are well to used [11].

3.8 Enhanced Algorithm

An enhanced ant colony algorithm identifies the best sequences for the different set of jobs. It is suggested that the optimization procedure outperforms the heuristics in the optimal solution. A new enhanced ant colony algorithm has been proposed where pheromone technique has been applied to given problem for generating optimum schedules. In future, the procedure could be tested for large scheduling problems with more objective functions [14].

4 ACO Applications

The ant colony optimization algorithms has been applied to many optimization problems like from travelling salesman problem, assignment problem, scheduling problem, routing problem and other combinatorial optimization problems.

4.1 Based on Travelling Salesman Problems

Dorigo *et al.* developed an ant system approach in the problem of travelling salesman in 1991. The first ACO algorithm was called the Ant System (AS) in which the author found that the shortest path to trip in the series of cities. The algorithm is relatively simple and based on a set of ants and each making one of the possible round-trips along the cities. The different algorithms were developed like Ant-Q and Min-Max AS in 1995 and 1997, respectively [19-23].

4.2 Based on Assignment Problems

Maniezzo, Colorni *et al.* developed first Ant System-Quadratic Assignment Problem (AS-QAP) in 1994; HAS-QAP and Min-Max QAP introduced by Gambardella *et al.* in 1997. Other developed problems are Generalized Assignment Problem (GAP), Frequency Assignment Problem (FAP) and Redundancy Assignment Problem (RAP) [24-27].

4.3 Based on Scheduling Problems

In 1994, firstly Colorni *et al* .were developed AS-Job Scheduling Problem (AS-JSP); AS-Flow Shop Problem (AS-FSP) was introduced by T. Stutzle in 1998 and D. Markle *et al.* were worked on ACO for resource-constrained project scheduling (ACO-RCPS) in the year of 2000. A train scheduling problem is also solved using ACO technique by K. Sankar in 2008 [5, 28-30].

4.4 Based on Routing Problems

Ant System-Vehicle Routing Problem (AS-VRP) was developed by Bullnheimer, Hartl *et al.* in 1997; a multiple ant colony system was introduced for VRP by L. M. Gambardella *et al.* in 1999. Further, a hybrid ACO algorithm for the extended capacitated arc routing problem by Li-Ning Xing *et al.*, routing in wireless sensor networks using an ACO router chip [15, 31-33].

4.5 Based on Other Applications

An improved ACO technique by using fuzzy inference rules for image classification and analysis, advances of soft computing methods in edge detection, an ACO approach for single machine tradiness problem, ant colony approach for makespan minimization on unrelated parallel machines, an ant colony-based framework for internet-scale peer-to-peer grids, urban traffic control system using modified ACO approach, a hybrid ACO approach for solving multi-objective design of optimization of air-cored solenoid, location area planning in cellular networks, forecast severe thunderstorms with optimum ranges of the stability indices, optimal transmission expansion planning, designing of PID controllers using ACO and designing of PID controller for a linear brushless DC motor [1, 4, 6, 9, 10, 11, 14, 16, 34-37].

5 Conclusion

As per the different classifications, ACO performs better against other global optimization techniques, retains memory of entire colony instead of previous generation only, less affected by poor initial solutions and can be used in dynamic applications and has been applied to a wide variety of applications. It is also a good choice for constrained discrete problems. Theoretical analysis is difficult; due to sequence of random decision, research is experimental rather than theoretical, convergence is guaranteed, but time to convergence is uncertain. In NP-hard problems, need high-quality solutions quickly-focus on quality of solutions and in dynamic network routing problems, need solutions for changing conditions. Coding though is somewhat complicated. Now a days, there are lot of upcoming applications in new trends related to soft computing, image edge detection, minimizing the error in parallel machine operation, traffic control system, cellular network planning and also in electric drive system like PID tuning; hopefully the more improved ACO approaches are going on with better convergence time and minimum margin of error.

References

[1] Andreas Bauer, Bernd Bullnheimer, Richard F. Hartl and Christine Straussl, "An Ant Colony Optimization Approach for the Single Machine Total Tradiness Problem", Proceeding of Congress on Evolutionary Computation, pp. 1-13, 1999.

[2] Thomas Stutzle and Holger H. Hoos, "MAX-MIN Ant System", Future Generation Computer Systems, pp. 889-914, 2000.

[3] John E. Bell and Patrick R. McMullen, "Ant colony optimization techniques for the vehicle routing problem", Advanced Engineering Informatics, Vol. 18, pp. 41-48, 2004.

[4] Francesco Palmieri, "An Ant Colony-based framework for Internet-scale Peer-to-Peer Grids", Journal of Information Technology and Applications, Vol.1 No. 4, pp. 249-260, 2007.

[5] K. Sankar, "Train Scheduling Using Ant Colony optimization Technique", Research Journal on Computer Engineering, Vol. 1, pp. 29-32, 2008.

[6] R. Foroughi, GH. A. Montazer and R. Sabzevari, "Design of a New urban Traffic Control system using Modified Ant Colony Optimization Approach", in Iranian Journal of science & Technology, Transaction B, Engineering, Vol. 32, No. B2, pp. 167-173, 2008.

[7] Kuo-Ling Huang and Ching-Jong Liao, "Ant colony optimization combined with taboo search for the job shop scheduling problem", Computers & Operations Research, pp. 1030-1046, 2008.

[8] Marco Dorigo and Thomas Stutzle, "Ant Colony Optimization: Overview and Recent Advances", Technical Report No. TR/IRIDIA/2009-013, pp. 1-32, 2009.

[9] Amir Atapour Abarghouei, Afshin Ghanizadeh and Siti Mariyam Shamsuddin, "Advances of Soft Computing Methods in Edge Detection", International Centre for Scientific Research and studies, Vol. 1, pp. 163-203, 2009.

[10] Waiel F. Abd El Wahed, A. A. Mousa, I. M. El-Desoky and R. M. Rizk-Allah, "A Hybrid Ant Colony optimization Approach for Solving Multiobjective Design optimization of Air-Cored Solenoid", The Online Journal on Power and Energy Engineering (OJPEE), Vol. (1) - No. (4), pp. 116-121, 2009.

[11] C. Naga Raju, O. Rama Devi, Sharda Mani and Sanam Nagendram, "An Improved Ant Colony Optimization Technique by using Fuzzy Inference Rules for Image Classification and Analysis", in International Journal of Advanced Engineering & Application, pp. 230-234, 2010.

[12] Mostafa Abd-El-Barr, "Ant Colony Heuristic Algorithm For Multi-Level Synthesis of Multiple-Valued Logic Functions", IAENG International Journal of Computer Science, Vol. 37, Issue 1, pp. 1-7, 2010.

[13] Vinay Chopra and Amardeep Singh, "Ant Colony Optimization approach for Solving FPGA routing with minimum Channel Width", International Journal on Computer Science and Engineering, Vol. 3, pp. 2855-2861, 2011.

[14] K. M. Senthil Kumar, Dr. V. Selladurai, Dr. K. Raja and Dr. K. ELANGOVAN, "Ant Colony Approach for Makespan Minimization on Unrelated Parallel Machines", in IJEST, Vol. 3 No. 4, pp. 3113-3120, 2011.

[15] Li-Ning Xing, Philipp Rohlfshagen, Ying-Wu Chen and Xin Yao, "A Hybrid Ant Colony optimization Algorithm for the Extended Capacitated Are Routing Problem", IEEE Transaction on systems, Man and Cybernetics, pp. 1-14, 2011.

[16] Ahmed Elwhishi, Issmail Ellabib and Idris. El-Feghi, "Ant Colony Optimization for Location Area Planning in Cellular Networks", web link: uqu.edu.sa/files2/tiny_mce/plugins//filemanager/files/30/…/F160.pdf.

[17] M. Brignone, G. Bozza, A. Randazzo, M. Piana and M. Pastorino, "A Hybrid Approach to 3D Microwave imaging by Using Linear Sampling and Ant Colony optimization", IEEE Transaction on Antennas and Propagation, Vol. 56, No. 10, pp. 3224-3232, 2008.

[18] B. Nagaraj, P. Vijayakumar, "A Comparative Study of PID Controller Tuning using GA, EP, PSO and ACO", Journal of Automation, Mobile Robotics & Intelligent Systems, Volume 5, pp. 42-48, 2011.

[19] M. Dorigo, "Optimization, Learning and Natural Algorithms", PhD thesis, Dipartimento di Elettronica, Politecnico di Milano, Italy, pp. 140, 1992.

[20] M. Dorigo, V. Maniezzo and A. Colorni, "The Ant System: An Autocatalytic Optimizing Process", Dipartimento di Eletronica, Politecnico di Milano, Italy, Technical Report 91-016, 1991.

[21] L. M. Gambardella and M. Dorigo, "Ant-Q: A reinforcement learning approach to the travelling salesman problem", in A. Prieditis and S. Russell, editors, Proceedings of the Twelfth International Conference on Machine Learning (ML-95), Morgan Kaufmann Publishers, polo Alto, CA, pp. 252-260, 1995.

[22] T. Stutzle, "Local Search Algorithms for Combinatorial Problems: Analysis, Improvements and New Applications", Infix, Sankt Augustin, Germany, 1999.

[23] T. Stutzle and H. H. Hoos, "The MAX-MIN Ant system and local search for the travelling salesman problem", Proceeding of the 1997 IEEE International Conference on Evolutionary Computation (ICEC'97), pp. 309-314, 1997.

[24] M. Dorigo and V. Maniezzo, "Parllel genetic algorithms: Introduction and overview of current research", In J. Stenders, editor, Parallel Genetic Algorithms: Theory and Applications, pp. 5-42, The Netherlands, 1992.

[25] M. Dorigo, V. Maniezzo and A. Colorni, " Positive feedback as a search strategy", Dipartimento di Elettronica, Politecnico di Milano, Italy, technical Report 91-016, 1991.

[26] M. Dorigo, V. Maniezzo and A. Colorni, "The Ant System: Optimization by a colony of cooperating agents", IEEE Transactions on Systems, Man and Cybernetics – Part B, 26(1): 29-41, 1996.

[27] M. Dorigo, M. Middendorf and T. Stutzle, editors, Abstract proceedings of ANTS200 – From Ant Colonies to Artificial Ants: A Series of International Workshops on Ant Algorithms, 7-9 September, 2000.

[28] A. Colorni, M. Dorigo, V. Maniezzo and M. Trubian, "Ant System for job-scheduling", JORBEL- Belgian Journal of Operation Research, Statistics and Computer Science, 34(1): 39-53, 1994.

[29] T. Stutzle, "An ant approach to the flow shop problem", in Proceedings of the 6th European Congress on Intelligent Techniques & Soft Computing (EU-FIT'98), Volume 3, pp. 1560-1564, 1998.

[30] D. Merkle, M. Middendorf and H. Schmeck, "Ant colony optimization for resource-constrained project scheduling", in Proceedings of the Genetic and Evolutionary Computation conference (GECCO-2000), pp. 893-900, Morgan Kaufmann Publishers, San Francisco, CA, 2000.

[31] B. Bullnheimer, R. F. Hartl and C. Strauss, "An Improved Ant System Algorithm for the Vehicle Routing Problem", Annals of Operations Research, 89:319-328, 1999.

[32] L. M. Gambardella, E. D. Taillard and G. Agazzi, "MACS-VRPTW: A multiple ant colony system for vehicle routing problems with time windows", in D. Corne, M. Dorigo and F. Glover, editors, New Ideas in Optimization, McGraw Hill, London, UK, pp. 63-76, 1999.

[33] Selcuk Okdem and Dervis Karaboga, "Routing in Wireless Sensor Networks Using an Ant Colony Optimization Router Chip", Erciyes University, Engineering Faculty, Computer Engineering Department, Kayseri, TR, Turkey, Sensors 2009, 9, 909-921.

[34] Sutapa Chaudhuri, "Ant colony optimization technique to forecast severe thunderstorms with optimum ranges of the stability indices", Department of Atmospheric Sciences, University of Calcutta, India, pp. 1-32, 2008.

[35] N. Leeprechanon, P. Limsakul and S. Pothiya, "Optimal Transmission Expansion Planning Using Ant Colony Optimization", in the Journal of Sustainable Energy & Environment, Volume 1, pp. 71-76, 2010.

[36] Ying-Tung Hsiao, Cheng-Long Chuang and Cheng-Chih Chien, "Ant Colony Optimization for Designing of PID Controllers", in IEEE International Symposium on Computer Aided Control Systems Design Taipei, Taiwan, pp. 321-326, September, 2004.

[37] N. Navidi, M. Bavafa and S. Hesami, "A New Approach for Designing of PID Controller for a Linear Brushless DC Motor with Using Ant Colony Search Algorithm", 978-1-4244-2487-0/09/$25.00 ©2009 IEEE.